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SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 22224

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		**Attempt any FIVE of the following:**	**10**
	a)	State whether the function $f(x)=\dfrac{e^x+e^{-x}}{2}$ is odd or even.	**02**
	Ans	$f(x)=\dfrac{e^x+e^{-x}}{2}$	
		$\therefore f(-x)=\dfrac{e^{-x}+e^{-(-x)}}{2}$	½
		$=\dfrac{e^{-x}+e^{x}}{2}$	½
		$=f(x)$	½
		$\therefore$ function is even.	½
	b)	If $f(x)=\dfrac{x^2+1}{x^3-1}$ find $f\left(\dfrac{1}{2}\right)$	**02**
	Ans	$f(x)=\dfrac{x^2+1}{x^3-1}$	
		$\therefore f\left(\dfrac{1}{2}\right)=\dfrac{\left(\dfrac{1}{2}\right)^2+1}{\left(\dfrac{1}{2}\right)^3-1}$	1
		$=\dfrac{-10}{7}$ OR -1.429	1

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Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	c)	Find $\frac{dy}{dx}$, if $y=\left(x^2+1\right)^5$	**02**
	Ans	$y=\left(x^2+1\right)^5$	
		$\therefore \frac{dy}{dx}=5\left(x^2+1\right)^4 . \frac{d}{dx}\left(x^2+1\right)$	
		$=5\left(x^2+1\right)^4 .(2x)$	1
		$=10x\left(x^2+1\right)^4$	1
	d)	Evaluate $\int(\tan x+\cot x)^2\ dx$	**02**
	Ans	$\int(\tan x+\cot x)^2\ dx$	
		$=\int\left(\tan^2 x+2\tan x\cot x+\cot^2 x\right)dx$	½
		$=\int\left(\tan^2 x+2+\cot^2 x\right)dx$	
		$=\int\left[\left(\sec^2 x-1\right)+2+\left(co\sec^2 x-1\right)\right]dx$	½
		$=\int\left(\sec^2 x+co\sec^2 x\right)dx$	
		$=\tan x-\cot x+c$	½ +½
	e)	Evaluate $\int \log x dx$	**02**
	Ans	$\int \log x dx=\int \log x\cdot 1\ dx$	½
		$=\log x\int 1dx-\int\left(\int 1dx\ \frac{d}{dx}\log x\right)dx$	½
		$=\log x(x)-\int x\ \frac{1}{x}dx$	½
		$=x\log x-\int 1dx$	
		$=x\log x-x+c$	½
		$=x(\log x-1)+c$	
	f)	Find the area between the lines $y=3x$, x-axis and the ordinates $x=1$ and $x=5$	**02**
	Ans	Area $A=\int_a^b y dx$	
		$=\int_1^5 3x dx$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	f)	$=3\int_1^5 xdx$	
		$=3\left[\frac{x^2}{2}\right]_1^5$	½
		$=3\left[\frac{5^2}{2}-\frac{1^2}{2}\right]$	½
		$=36$	½
		--	
	g)	Show that there exist a root of the equation $x^2-2x-1=0$ in $(-1,0)$ and find approximate value of the root by using Bisection method. $(\text{Use two iterations})$	**02**
	Ans	$x^2-2x-1=0$	
		$f(x)=x^2-2x-1$	
		$f(-1)=2$	
		$f(0)=-1$	½
		root is in $(-1,0)$	
		$\therefore x_1=\frac{-1+0}{2}=-0.5$	½
		$\therefore f(-0.5)=0.25$	½
		$\therefore$ root is in $(-0.5,0)$	
		$\therefore x_2=\frac{-0.5+0}{2}=-0.25$	½
		OR	
		$x^2-2x-1=0$	
		$f(x)=x^2-2x-1$	
		$f(-1)=2$	
		$f(0)=-1$	
		root is in $(-1,0)$	

a	b	$x=\frac{a+b}{2}$	$f(x)$
-1	0	-0.5	0.25
-0.5	0	-0.25	-----

Marking scheme: 1 (first row), 1 (second row)

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.		**Attempt any THREE of the following:**	**12**
	a)	Find $\frac{dy}{dx}$ if $\cos\left(x^2+y^2\right)=\log\left(xy\right)$	**04**
	Ans	$\cos\left(x^2+y^2\right)=\log\left(xy\right)$	
		$\therefore -\sin\left(x^2+y^2\right)\left(2x+2y\frac{dy}{dx}\right)=\frac{1}{xy}\left(x\frac{dy}{dx}+y\right)$	2
		$\therefore -2x\sin\left(x^2+y^2\right)-2y\sin\left(x^2+y^2\right)\frac{dy}{dx}=\frac{1}{y}\frac{dy}{dx}+\frac{1}{x}$	
		$\therefore \frac{dy}{dx}\left(-2y\sin\left(x^2+y^2\right)-\frac{1}{y}\right)=\frac{1}{x}+2x\sin\left(x^2+y^2\right)$	1
		$\therefore \frac{dy}{dx}=\dfrac{\frac{1}{x}+2x\sin\left(x^2+y^2\right)}{-2y\sin\left(x^2+y^2\right)-\frac{1}{y}}$	1
		--	
	b)	If $x=a\cos^3\theta, y=a\sin^3\theta$, find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$	**04**
	Ans	$x=a\cos^3\theta$ $\quad$ $y=a\sin^3\theta$	
		$\therefore \frac{dx}{d\theta}=-3a\cos^2\theta\sin\theta$ $\quad$ $\frac{dy}{d\theta}=3a\sin^2\theta\left(\cos\theta\right)$	1
		$\therefore \frac{dy}{dx}=\dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	
		$\frac{dy}{dx}=\frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta}$	1
		$\frac{dy}{dx}=-\tan\theta$	1
		at $\theta=\frac{\pi}{4}$	
		$\frac{dy}{dx}=-\tan\frac{\pi}{4}$	
		$\frac{dy}{dx}=-1$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	c)	Find the maximum and minimum value of $2x^3-3x^2-36x+10$	**04**
	Ans	Let $y=2x^3-3x^2-36x+10$	
		$\therefore \frac{dy}{dx}=6x^2-6x-36$	½
		$\therefore \frac{d^2y}{dx^2}=12x-6$	½
		Consider $\frac{dy}{dx}=0$	
		$6x^2-6x-36=0$	½
		$x^2-x-6=0$	
		$\therefore x=-2\ ,\ x=3$	½
		at $x=-2$	
		$\frac{d^2y}{dx^2}=12(-2)-6=-30<0$	½
		$\therefore y$ is maximum at $x=-2$	
		$y_{\max}=2(-2)^3-3(-2)^2-36(-2)+10$ $=54$	½
		$at\ \ x=3$	
		$\frac{d^2y}{dx^2}=12(3)-6=30>0$	½
		$\therefore y$ is minimum at $x=3$	
		$y_{\min}=2(3)^3-3(3)^2-36(3)+10$ $=-71$	½
		--	
	d)	A beam is bent in the form of the curve $y=2\sin x-\sin 2x$ Find the radius of curvature of the beam at the point $x=\frac{\pi}{2}$	**04**
	Ans	$y=2\sin x-\sin 2x$	
		$\therefore \frac{dy}{dx}=2\cos x-2\cos 2x$	½
		$\therefore \frac{d^2y}{dx^2}=-2\sin x+4\sin 2x$	½
		$\therefore$ at $x=\frac{\pi}{2}$	
		$\frac{dy}{dx}=2\cos\left(\frac{\pi}{2}\right)-2\cos 2\left(\frac{\pi}{2}\right)=2$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	d)	$\frac{d^2y}{dx^2}=-2\sin\left(\frac{\pi}{2}\right)+4\sin 2\left(\frac{\pi}{2}\right)=-2$	½
		$\therefore$ Radius of curvature is $\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	
		$\therefore \rho=\frac{\left[1+(2)^2\right]^{\frac{3}{2}}}{-2}$	1
		$\therefore \rho=-5.590$ or 5.590	1
		--	
3.		**Attempt any <u>THREE</u> of the following:**	**12**
	a)	Find the points on the curve $y=x^3+3x^2-9x+7$ at which tangents drawn are parallel to $x-$axis.	**04**
	Ans	$y=x^3+3x^2-9x+7$	
		$\frac{dy}{dx}=3x^2+6x-9$	1
		$\because$ tangent is parallel to x-axis	
		$\therefore$ slope of tangent = slope of x-axis	
		$\therefore \frac{dy}{dx}=0$	1
		$\therefore 3x^2+6x-9=0$	
		$\therefore x=1$; $x=-3$	
		$\therefore y=2$; $y=34$	1
		$\therefore$ points are $(1,2)$ and $(-3,34)$	1
		--	
	b)	Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$	**04**
	Ans	Let $u=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ and $v=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$	
		Put $x=\tan\theta \Rightarrow \tan^{-1}x=\theta$	
		$u=\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$	½
		$u=\tan^{-1}(\tan 2\theta)$	½
		$u=2\theta$	
		$u=2\tan^{-1}x$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	b)	$\frac{du}{dx}=\frac{2}{1+x^2}$	½
		$v=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$	
		$v=\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$	½
		$v=\sin^{-1}(\sin 2\theta)$	½
		$v=2\theta$	
		$v=2\tan^{-1}x$	
		$\frac{dv}{dx}=\frac{2}{1+x^2}$	½
		$\therefore \frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{\frac{2}{1+x^2}}{\frac{2}{1+x^2}}$	½
		$\therefore \frac{du}{dv}=1$	½
		OR	
		Let $u=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$	
		$\therefore \frac{du}{dx}=\frac{1}{1+\left(\frac{2x}{1-x^2}\right)^2}\times\left[\frac{(1-x^2)2-2x(-2x)}{(1-x^2)^2}\right]$	1
		$\therefore \frac{du}{dx}=\frac{(1-x^2)^2}{(1-x^2)^2+4x^2}\left[\frac{2+2x^2}{(1-x^2)^2}\right]$	
		$\therefore \frac{du}{dx}=\frac{2+2x^2}{(1-x^2)^2+4x^2}$	
		$\therefore \frac{du}{dx}=\frac{2(1+x^2)}{(1-x^2)^2+4x^2}$	
		$\therefore \frac{du}{dx}=\frac{2(1+x^2)}{(1+x^2)^2}=\frac{2}{1+x^2}$	½
		$\therefore v=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	b)	$\therefore \frac{dv}{dx}=\frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}}\times\left[\frac{\left(1+x^2\right)2-2x\left(2x\right)}{\left(1+x^2\right)^2}\right]$	1
		$\therefore \frac{dv}{dx}=\frac{\left(1+x^2\right)}{\sqrt{\left(1+x^2\right)^2-4x^2}}\left[\frac{2-2x^2}{\left(1+x^2\right)^2}\right]$	
		$\therefore \frac{dv}{dx}=\frac{\left(2-2x^2\right)}{\left(1+x^2\right)\sqrt{\left(1+x^2\right)^2-4x^2}}$	
		$\therefore \frac{dv}{dx}=\frac{2\left(1-x^2\right)}{\left(1+x^2\right)\left(1-x^2\right)}$	
		$\therefore \frac{dv}{dx}=\frac{2}{\left(1+x^2\right)}$	½
		$\therefore \frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{\frac{2}{\left(1+x^2\right)}}{\frac{2}{\left(1+x^2\right)}}$	½
		$\therefore \frac{du}{dv}=1$	½
		--	
	c)	Find $\frac{dy}{dx}$ if $y=\left(\log x\right)^x+x^{\cos^{-1}x}$	**04**
	Ans	Let $u=\left(\log x\right)^x$	
		$\log u=x\log\left(\log x\right)$	½
		$\frac{1}{u}\frac{du}{dx}=x\frac{1}{\log x}\frac{1}{x}+\log\left(\log x\right)$	1
		$\therefore \frac{du}{dx}=u\left(\frac{1}{\log x}+\log\left(\log x\right)\right)$	
		$\therefore \frac{du}{dx}=\left(\log x\right)^x\left[\frac{1}{\log x}+\log\left(\log x\right)\right]$	½
		Let $v=x^{\cos^{-1}x}$	
		$\log v=\cos^{-1}x\log x$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	c)	$\frac{1}{v}\frac{dv}{dx}=\cos^{-1}x\left(\frac{1}{x}\right)+\log x\left(\frac{-1}{\sqrt{1-x^2}}\right)$	½
		$\therefore \frac{dv}{dx}=x^{\cos^{-1}x}\left[\left(\cos^{-1}x\right)\left(\frac{1}{x}\right)-\log x\left(\frac{1}{\sqrt{1-x^2}}\right)\right]$	½
		$\therefore \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$	
		$\therefore \frac{dy}{dx}=\left(\log x\right)^x\left[\frac{1}{\log x}+\log\left(\log x\right)\right]+x^{\cos^{-1}x}\left[\left(\cos^{-1}x\right)\left(\frac{1}{x}\right)-\log x\left(\frac{1}{\sqrt{1-x^2}}\right)\right]$	½
		--	
	d)	Evaluate: $\int\frac{\sec x\cos ecx}{\log\tan x}\,dx$	**04**
	Ans	$\int\frac{\sec x\cos ecx}{\log\tan x}\,dx$	
		Put $\log\tan x=t$	1
		$\therefore \frac{1}{\tan x}\sec^2 xdx=dt$	
		$\therefore \frac{\cos x}{\sin x}\frac{1}{\cos^2 x}dx=dt$	
		$\therefore \sec x\cos ecxdx=dt$	1
		$=\int\frac{1}{t}\,dt$	½
		$=\log t+c$	1
		$=\log\left(\log\left(\tan x\right)\right)+c$	½
		--	
4.		**Attempt any THREE of the following:**	**12**
	a)	Evaluate : $\int\frac{1}{2x^2+3x+1}dx$	**04**
	Ans	$\int\frac{1}{2x^2+3x+1}dx$	
		$=\frac{1}{2}\int\frac{1}{x^2+\frac{3}{2}x+\frac{1}{2}}dx$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	a)	Third term $=\left(\frac{1}{2}\times\frac{3}{2}\right)^2=\frac{9}{16}$	
		$=\frac{1}{2}\int\frac{1}{x^2+\frac{3}{2}x+\frac{9}{16}-\frac{9}{16}+\frac{1}{2}}dx$	1
		$=\frac{1}{2}\int\frac{1}{\left(x+\frac{3}{4}\right)^2-\left(\frac{1}{4}\right)^2}dx$	1
		$=\frac{1}{2}\left[\frac{1}{2\left(\frac{1}{4}\right)}\log\left(\frac{x+\frac{3}{4}-\frac{1}{4}}{x+\frac{3}{4}+\frac{1}{4}}\right)\right]+c$	1½
		$=\log\left(\frac{2x+1}{2x+2}\right)+c$	
		OR	
		$\int\frac{1}{2x^2+3x+1}dx=\int\frac{1}{(2x+1)(x+1)}dx$	½
		Let $\frac{1}{(2x+1)(x+1)}=\frac{A}{2x+1}+\frac{B}{x+1}$	
		$1=A(x+1)+B(2x+1)$	
		Put $x=\frac{-1}{2}$	
		$\therefore A=2$	½
		Put $x=-1$	
		$\therefore B=-1$	½
		$\frac{1}{(2x+1)(x+1)}=\frac{2}{2x+1}+\frac{-1}{x+1}$	½
		$\int\frac{1}{(2x+1)(x+1)}dx=\int\left(\frac{2}{2x+1}+\frac{-1}{x+1}\right)dx$	
		$=\frac{2\log(2x+1)}{2}-\log(x+1)+c$	1+1
		$=\log(2x+1)-\log(x+1)+c$	
		OR	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	a)	$\int \frac{1}{2x^2+3x+1}dx$	
		Third term $=\frac{(M.T.)^2}{4(F.T.)}=\frac{9}{8}$	½
		$=\int \frac{1}{2x^2+3x+\frac{9}{8}-\frac{9}{8}+1}dx$	1
		$=\int \frac{1}{\left(\sqrt{2}x+\frac{3}{\sqrt{8}}\right)^2-\frac{1}{8}}dx$	1
		$=\int \frac{1}{\left(\sqrt{2}x+\frac{3}{\sqrt{8}}\right)^2-\left(\frac{1}{\sqrt{8}}\right)^2}dx$	½
		$=\frac{1}{\sqrt{2}}\left[\frac{1}{2\left(\frac{1}{\sqrt{8}}\right)}\log\left(\frac{\sqrt{2}x+\frac{3}{\sqrt{8}}-\frac{1}{\sqrt{8}}}{\sqrt{2}x+\frac{3}{\sqrt{8}}+\frac{1}{\sqrt{8}}}\right)\right]+c$	1
		$=\log\left(\frac{2x+1}{2x+2}\right)+c$	
		--	
			04
	b)	Evaluate : $\int \frac{dx}{a^2\sin^2 x+b^2\cos^2 x}$	
	Ans	$\int \frac{dx}{a^2\sin^2 x+b^2\cos^2 x}$	
		$=\int \frac{dx/\cos^2 x}{\frac{a^2\sin^2 x+b^2\cos^2 x}{\cos^2 x}}$	½
		$=\int \frac{\sec^2 x dx}{a^2\tan^2 x+b^2}$ Put $\tan x=t$, $\therefore \sec^2 x dx=dt$	½
		$=\int \frac{dt}{a^2 t^2+b^2}$	1
		$=\int \frac{dt}{a^2\left(t^2+\frac{b^2}{a^2}\right)}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	b)	$=\frac{1}{a^2}\int\frac{dt}{t^2+\left(\frac{b}{a}\right)^2}=\frac{1}{a^2}\cdot\frac{1}{\frac{b}{a}}\tan^{-1}\left(\frac{t}{\frac{b}{a}}\right)+c$	1
		$=\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)+c$	½
		OR	
		$\int\frac{dx}{a^2\sin^2x+b^2\cos^2x}$	
		$=\int\frac{dx/\cos^2x}{\frac{a^2\sin^2x+b^2\cos^2x}{\cos^2x}}$	½
		$=\int\frac{\sec^2 xdx}{a^2\tan^2x+b^2}$ $Put\ \tan x=t$, $\therefore\sec^2 xdx=dt$	½
		$=\int\frac{dt}{a^2t^2+b^2}$	1
		$=\frac{1}{b}\tan^{-1}\left(\frac{at}{b}\right)\frac{1}{a}+c$	1½
		$=\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)+c$	½
		--	
	c)	Evaluate : $\int x\cos ec^{-1}xdx$	**04**
	Ans	$\int x\cos ec^{-1}xdx$	
		$=\cos ec^{-1}x\int xdx-\int\left(\int xdx\ \frac{d}{dx}\cos ec^{-1}x\right)dx$	½
		$=\cos ec^{-1}x\cdot\frac{x^2}{2}-\int\frac{x^2}{2}\cdot\left(\frac{-1}{x\sqrt{x^2-1}}\right)\cdot dx$	1
		$=\cos ec^{-1}x\cdot\frac{x^2}{2}+\frac{1}{2}\int\frac{x}{\sqrt{x^2-1}}\cdot dx$	
		$=\cos ec^{-1}x\cdot\frac{x^2}{2}+\frac{1}{4}\int\frac{2x}{\sqrt{x^2-1}}\cdot dx$	1
		$=\cos ec^{-1}x\cdot\frac{x^2}{2}+\frac{1}{4}\left(2\sqrt{x^2-1}\right)+c$	1
		$=\cos ec^{-1}x\cdot\frac{x^2}{2}+\frac{1}{2}\left(\sqrt{x^2-1}\right)+c$	½
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	d)	Evaluate : $\int \frac{1}{x(2-\log x)(2\log x-1)}dx$	**04**
	Ans	$\int \frac{1}{x(2-\log x)(2\log x-1)}dx$ Put $\log x = t$, $\therefore \frac{1}{x}dx = dt$	½
		$\int \frac{1}{(2-t)(2t-1)}dt$	½
		$\frac{1}{(2-t)(2t-1)} = \frac{A}{2-t}+\frac{B}{2t-1}$	
		$1 = A(2t-1)+B(2-t)$	
		$\therefore$ Put $t=2$, $A=\frac{1}{3}$	½
		Put $t=\frac{1}{2}$, $B=\frac{2}{3}$	½
		$\therefore \frac{1}{(2-t)(2t-1)} = \frac{\frac{1}{3}}{2-t}+\frac{\frac{2}{3}}{2t-1}$	
		$\int \frac{1}{(2-t)(2t-1)}dt = \int\left(\frac{\frac{1}{3}}{2-t}+\frac{\frac{2}{3}}{2t-1}\right)dt$	½
		$=-\frac{1}{3}\log[2-t]+\frac{2}{6}\log[2t-1]+c$	1
		$=-\frac{1}{3}\log[2-\log x]+\frac{1}{3}\log[2\log x-1]+c$	½
		OR	
		$\int \frac{1}{x(2-\log x)(2\log x-1)}dx$ Put $\log x = t$, $\therefore \frac{1}{x}dx = dt$	½
		$\int \frac{1}{(2-t)(2t-1)}dt$	
		$=\int \frac{1}{-2t^2+5t-2}dt$	
		$=\frac{-1}{2}\int \frac{1}{t^2-\frac{5}{2}t+1}dt$	½

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4.	d)	$=\frac{-1}{2}\int\frac{1}{t^2-\frac{5}{2}t+\frac{25}{16}-\frac{25}{16}+1}dt$	½
		$=\frac{-1}{2}\int\frac{1}{\left(t-\frac{5}{4}\right)^2-\left(\frac{3}{4}\right)^2}dt$	1
		$=\frac{-1}{2}\frac{1}{2\frac{3}{4}}\log\left\|\frac{t-\frac{5}{4}-\frac{3}{4}}{t-\frac{5}{4}+\frac{3}{4}}\right\|+c$	1
		$=\frac{-1}{3}\log\left\|\frac{t-2}{t-\frac{1}{2}}\right\|+c$	
		$=\frac{-1}{3}\log\left\|\frac{\log x-2}{\log x-\frac{1}{2}}\right\|+c$	½
		---	
	e)	Evaluate: $\int_1^4\frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+4}}\,dx$	**04**
	Ans	$I=\int_1^4\frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+4}}\,dx$ --------------------(1)	
		$I=\int_1^4\frac{\sqrt[3]{9-(5-x)}}{\sqrt[3]{9-(5-x)}+\sqrt[3]{(5-x)+4}}\,dx$	1
		$\therefore I=\int_1^4\frac{\sqrt[3]{x+4}}{\sqrt[3]{x+4}+\sqrt[3]{9-x}}\,dx$--------------------(2)	
		add (1) and (2), $I+I=\int_1^4\frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+4}}\,dx+\int_1^4\frac{\sqrt[3]{x+4}}{\sqrt[3]{x+4}+\sqrt[3]{9-x}}\,dx$	
		$\therefore 2I=\int_1^4\frac{\sqrt[3]{9-x}+\sqrt[3]{x+4}}{\sqrt[3]{9-x}+\sqrt[3]{x+4}}\,dx$	1
		$\therefore 2I=\int_1^4 1\,dx$	
		$\therefore 2I=(x)_1^4$	1
		$\therefore I=\frac{3}{2}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.		**Attempt any TWO of the following:**	**12**
	a)	Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{9}+\frac{y^2}{4}$ about $x-$axis	**06**
	Ans	Consider $\frac{x^2}{9}+\frac{y^2}{4}=1$	
		$\therefore y^2=\frac{4}{9}\left(9-x^2\right)$	1
		Volume of solid $V=\pi\int_{-a}^{a} y^2dx$	
		$V=\pi\int_{-3}^{3}\frac{4}{9}\left(9-x^2\right)dx$	1
		$\therefore V=2\pi\int_{0}^{3}\frac{4}{9}\left(9-x^2\right)dx$	1
		$\therefore V=\frac{8\pi}{9}\left[9x-\frac{x^3}{3}\right]_0^3$	1
		$\therefore V=\frac{8\pi}{9}\left[\left(9(3)-\frac{3^3}{3}\right)-\left(9(0)-\frac{0^3}{3}\right)\right]$	1
		$V=16\pi$	1
		(Note :If student has considered/assumed other value than 1and attempted to solve the problem , give appropriate marks.)	
		--	
	b)	**Attempt the following:**	**06**
	(i)	Form the diffrential equation by eliminating the arbitrary constants if $y= a\cos\left(\log x\right)+b\sin\left(\log x\right)$	**03**
	Ans	$y= a\cos\left(\log x\right)+b\sin\left(\log x\right)$	
		$\therefore \frac{dy}{dx}=-a\sin\left(\log x\right)\frac{1}{x}+b\cos\left(\log x\right)\frac{1}{x}$	1
		$\therefore x\frac{dy}{dx}=-a\sin\left(\log x\right)+b\cos\left(\log x\right)$	
		$\therefore x\frac{d^2y}{dx^2}+\frac{dy}{dx}=-a\cos\left(\log x\right)\frac{1}{x}-b\sin\left(\log x\right)\frac{1}{x}$	1
		$\therefore x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=-\left(a\cos\left(\log x\right)+b\sin\left(\log x\right)\right)$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	b)(i)	$\therefore x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}=-y$	½
		$\therefore x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+y=0$	
	b)(ii)	Solve the differential equation: $\dfrac{dy}{dx}+y\tan x=\cos^2 x$	**03**
	Ans	$\dfrac{dy}{dx}+y\tan x=\cos^2 x$ Comparing with $\dfrac{dy}{dx}+Py=Q$	½
		$\therefore P=\tan x$ and $Q=\cos^2 x$	
		$IF=e^{\int\tan x dx}=e^{\log(\sec x)}=\sec x$	½
		$\therefore y\cdot IF=\int Q\cdot IF dx+c$	
		$y\cdot\sec x=\int\cos^2 x\sec x dx+c$	1
		$y\cdot\sec x=\int\cos x dx+c$	
		$y\cdot\sec x=\sin x+c$	1
	c)	In a single closed electrical circuit the current ' I ' at time t is given by $E-RI-L\dfrac{dI}{dt}=0$. Find the current I at time t, given that t=0 , I=0 and L,R,E are constants.	**06**
	Ans	$E-RI-L\dfrac{dI}{dt}=0$	
		$\therefore\dfrac{dI}{dt}+\dfrac{R}{L}I=\dfrac{E}{L}$ Comparing with $\dfrac{dy}{dx}+Py=Q$	½
		$\therefore P=\dfrac{R}{L}$ and $Q=\dfrac{E}{L}$	
		$IF=e^{\int\frac{R}{L}dt}=e^{\frac{R}{L}t}$	1
		$\therefore I\cdot IF=\int Q\cdot IF dt+c$	
		$I\cdot e^{\frac{R}{L}t}=\int\dfrac{E}{L}e^{\frac{R}{L}t}dt+c$	1
		$I\cdot e^{\frac{R}{L}t}=\dfrac{E}{L}\dfrac{e^{\frac{R}{L}t}}{\frac{R}{L}}+c$	1
		$I\cdot e^{\frac{R}{L}t}=\dfrac{E}{R}e^{\frac{R}{L}t}+c$	
		When $t=0$, $\mathrm{I}=0$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	$\therefore c=-\dfrac{E}{R}$	1
		$I\cdot e^{\frac{R}{L}t}=\dfrac{E}{R}e^{\frac{R}{L}t}-\dfrac{E}{R}$	½
		$I=\dfrac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$	1
		--	
6.		**Attempt any <u>TWO</u> of the following:**	**12**
	a)	**Attempt the following:**	**06**
	(i)	Solve the following system of by equtions by Jacobi's -Iteration method. (Two iterations) $5x+2y+z=12$, $x+4y+2z=15$, $x+2y+5z=20$	**03**
	Ans	$x=\dfrac{1}{5}(12-2y-z)$ $y=\dfrac{1}{4}(15-x-2z)$ $z=\dfrac{1}{5}(20-x-2y)$	1
		Starting with $x_0=y_0=z_0=0$ $x_1=2.4$ $y_1=3.75$ $z_1=4$	1
		$x_2=0.1$ $y_2=1.15$ $z_2=2.02$	1
		--	
	a(ii)	Solve the following system of equation by using Gauss-Seidel method. (Two iterations) $15x+2y+z=18$, $2x+20y-3z=19$, $3x-6y+25z=22$	**03**
	Ans	$x=\dfrac{1}{15}(18-2y-z)$ $y=\dfrac{1}{20}(19-2x+3z)$ $z=\dfrac{1}{25}(22-3x+6y)$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	a)(ii)	Starting with $y_0 = z_0 = 0$ $x_1 = 1.2$ $y_1 = 0.83$ $z_1 = 0.935$	1
		$x_2 = 1.027$ $y_2 = 0.988$ $z_2 = 0.994$	1
		--	
	b)	Solve the following system of equations by using Gauss-elimination method $6x - y - z = 19$, $3x + 4y + z = 26$, $x + 2y + 6z = 22$	**06**
	Ans	$6x - y - z = 19$ $3x + 4y + z = 26$ $x + 2y + 6z = 22$	
		$6x - y - z = 19$; $3x + 4y + z = 26$; + ; $9x + 3y = 45$ and $18x + 24y + 6z = 156$; $x + 2y + 6z = 22$; − ; $17x + 22y = 134$	1+1
		$\therefore\ 3x + y = 15$ $66x + 22y = 330$ $17x + 22y = 134$ − $49x = 196$	1
		$\therefore x = 4$	1
		$y = 3$	1
		$z = 2$	1
		--	
		Note: *In the above solution, first z is eliminated and then y is eliminated to find the value of x first. If in case the problem is solved by elimination of another unknown i. e., either first x or y, appropriate marks to be given as per above scheme of marking.*	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	c)	Find the approximate root of the equation $x^4 - x - 10 = 0$, by Newton-Raphson method (Carry out four iterations)	
	Ans	Let $f(x) = x^4 - x - 10$	
		$f(1) = -10 < 0$	
		$f(2) = 4 > 0$	1
		$f'(x) = 4x^3 - 1$	1
		Initial root $x_0 = 2$	
		$\therefore f'(2) = 31$	
		$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)} = 1.871$	1
		$x_2 = 1.871 - \frac{f(1.871)}{f'(1.871)} = 1.856$	1
		$x_3 = 1.856 - \frac{f(1.856)}{f'(1.856)} = 1.856$	1
		$x_4 = 1.856 - \frac{f(1.856)}{f'(1.856)} = 1.856$	1
		OR	
		Let $f(x) = x^4 - x - 10$	
		$f(1) = -10 < 0$	
		$f(2) = 4 > 0$	1
		$f'(x) = 4x^3 - 1$	1
		Initial root $x_0 = 2$	
		$\therefore f'(2) = 31$	
		$x_i = \frac{xf'(x) - [f(x)]}{f'(x)} = \frac{x(4x^3 - 1) - [x^4 - x - 10]}{4x^3 - 1}$	
		$= \frac{3x^4 + 10}{4x^3 - 1}$	2
		$x_1 = 1.871$	½
		$x_2 = 1.856$	½
		$x_3 = 1.856$	½
		$x_4 = 1.856$	½
		OR	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	c)	Let $f(x)=x^4-x-10$	
		$f(-1)=-8<0$	
		$f(-2)=8>0$	1
		$f'(x)=4x^3-1$	1
		Initial root $x_0=-2$	
		$\therefore f'(-2)=-33$	
		$x_1=x_0-\frac{f(x_0)}{f'(x_0)}=-2-\frac{f(-2)}{f'(-2)}=-1.758$	1
		$x_2=-1.758-\frac{f(-1.758)}{f'(-1.758)}=-1.700$	1
		$x_3=-1.700-\frac{f(-1.700)}{f'(-1.700)}=-1.697$	1
		$x_4=-1.697-\frac{f(-1.697)}{f'(-1.697)}=-1.697$	1
		OR	
		Let $f(x)=x^4-x-10$	
		$f(-1)=-8<0$	
		$f(-2)=8>0$	1
		$f'(x)=4x^3-1$	1
		Initial root $x_0=-2$	
		$\therefore f'(-2)=-33$	
		$x_i=\frac{xf'(x)-[f(x)]}{f'(x)}=\frac{x(4x^3-1)-[x^4-x-10]}{4x^3-1}$	
		$=\frac{3x^4+10}{4x^3-1}$	2
		$x_1=-1.758$	½
		$x_2=-1.700$	½
		$x_3=-1.697$	½
		$x_4=-1.697$	½
		--	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
		Important Note *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.* -- --	

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Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		**Solve any FIVE of following:**	**10**
	a)	If $f(x)=x^3-x$, find $f(1)+f(2)$	**02**
	Ans	$f(x)=x^3-x$	
		$\therefore f(1)=(1)^3-(1)=0$	½
		$\therefore f(2)=(2)^3-(2)=6$	½
		$\therefore f(1)+f(2)=0+6$	½
		$\therefore f(1)+f(2)=6$	½
		--	
	b)	State whether the function $f(x)=x^3-3x+\sin x+x.\cos x$ is even or odd.	**02**
	Ans	$f(x)=x^3-3x+\sin x+x.\cos x$	
		$\therefore f(-x)=(-x)^3-3(-x)+\sin(-x)+(-x).\cos(-x)$	½
		$=-x^3+3x-\sin x-x.\cos x$	½
		$=-(x^3-3x+\sin x+x.\cos x)$	½
		$=-f(x)$	½
		$\therefore$ Function is odd.	
		--	
	c)	Find $\frac{dy}{dx}$ if $y=e^{2x}.\log(x+1)$	**02**

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Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	c)	$y = e^{2x}.\log(x+1)$	
	Ans	$\therefore \frac{dy}{dx} = e^{2x}.\frac{1}{x+1} + \log(x+1).e^{2x}.2$	1+1
		$= \frac{e^{2x}}{x+1} + 2e^{2x}\log(x+1)$	
		---	
	d)	Evaluate $\int\left(e^{2x} + \frac{1}{1+x^2}\right)dx$	**02**
	Ans	$\int\left(e^{2x} + \frac{1}{1+x^2}\right)dx$	
		$= \frac{e^{2x}}{2} + \tan^{-1}x + c$	1+1
		---	
	e)	Evaluate $\int\frac{dx}{9x^2-16}$	**02**
	Ans	$\int\frac{dx}{9x^2-16} = \frac{1}{9}\int\frac{dx}{x^2-\frac{16}{9}}$	½
		$= \frac{1}{9}\int\frac{dx}{x^2-\left(\frac{4}{3}\right)^2}$	
		$= \frac{1}{9}\frac{1}{2.\frac{4}{3}}\log\left(\frac{x-\frac{4}{3}}{x+\frac{4}{3}}\right)+c$	1
		$= \frac{1}{24}\log\left(\frac{3x-4}{3x+4}\right)+c$	½
		OR	
		$\int\frac{dx}{9x^2-16} = \int\frac{dx}{(3x)^2-(4)^2}$	
		$= \frac{1}{2(4)}.\frac{1}{3}\log\left(\frac{3x-4}{3x+4}\right)+c$	1
		$= \frac{1}{24}\log\left(\frac{3x-4}{3x+4}\right)+c$	1

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Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	f)	Find the area enclosed by the curve $y = x^3$,x-axis and the ordinates $x=1$ and $x=3$	**02**
	Ans	Area $A=\int_a^b y\,dx$	
		$=\int_1^3 x^3 dx$	½
		$=\left[\frac{x^4}{4}\right]_1^3$	½
		$=\frac{(3)^4}{4}-\frac{(1)^4}{4}$	½
		$=\frac{81}{4}-\frac{1}{4}$	
		$=20$	½
	g)	Show that the root of $x^3-9x+1=0$ lies between 2 and 3.	**02**
	Ans	$f(x)=x^3-9x+1$	
		$f(2)=(2)^3-9(2)+1=-9<0$	1
		$f(3)=(3)^3-9(3)+1=1>0$	1
		$\therefore$ Root lies between 2 and 3	
2.		**Solve any THREE of the following:**	**12**
	a)	If $x^2+y^2+2xy-y=0$ find $\frac{dy}{dx}$ at $(1,2)$	**04**
	Ans	$x^2+y^2+2xy-y=0$	
		$2x+2y\frac{dy}{dx}+2\left(x\frac{dy}{dx}+y\right)-\frac{dy}{dx}=0$	2
		$2x+2y\frac{dy}{dx}+2x\frac{dy}{dx}+2y-\frac{dy}{dx}=0$	
		$2y\frac{dy}{dx}+2x\frac{dy}{dx}-\frac{dy}{dx}=-2x-2y$	
		$(2y+2x-1)\frac{dy}{dx}=-2x-2y$	
		$\frac{dy}{dx}=\frac{-2x-2y}{2y+2x-1}=\frac{-2(x+y)}{2y+2x-1}$	1
		$\left(\frac{dy}{dx}\right)_{(1,2)}=\frac{-2(1+2)}{2(2)+2(1)-1}=\frac{-6}{5}$ OR -1.2	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	b)	If $x=a\left(2\theta-\sin 2\theta\right), y=a\left(1-\cos 2\theta\right)$ find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$	**04**
	Ans	$x=a\left(2\theta-\sin 2\theta\right)$ $\quad$ $y=a\left(1-\cos 2\theta\right)$	
		$\therefore \frac{dx}{d\theta}=a\left(2-2\cos 2\theta\right)$ $\quad$ $\frac{dy}{d\theta}=2a\sin 2\theta$	1+1
		$\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	
		$\therefore \frac{dy}{dx}=\frac{2a\sin 2\theta}{2a\left(1-\cos 2\theta\right)}=\frac{\sin 2\theta}{\left(1-\cos 2\theta\right)}$ OR $\frac{dy}{dx}=\frac{\sin 2\theta}{\left(1-\cos 2\theta\right)}=\frac{2\sin\theta\cos\theta}{2\sin^2\theta}=\cot\theta$	1
		at $\theta=\frac{\pi}{4}$	
		$\therefore \frac{dy}{dx}=\frac{\sin 2\left(\frac{\pi}{4}\right)}{\left(1-\cos 2\left(\frac{\pi}{4}\right)\right)}=\frac{\sin\left(\frac{\pi}{2}\right)}{\left(1-\cos\left(\frac{\pi}{2}\right)\right)}$	
		$\therefore \frac{dy}{dx}=\frac{1}{1-0}=1$ OR $\frac{dy}{dx}=\cot\frac{\pi}{4}=1$	1
		--	
	c)	Find the maximum and minimum value of $y=x^3-\frac{15}{2}x^2+18x$	**04**
	Ans	Let $y=x^3-\frac{15}{2}x^2+18x$	
		$\therefore \frac{dy}{dx}=3x^2-15x+18$	½
		$\therefore \frac{d^2y}{dx^2}=6x-15$	½
		Consider $\frac{dy}{dx}=0$	
		$3x^2-15x+18=0$	½
		$x^2-5x+6=0$	
		$\therefore x=2$ or $x=3$	½
		at $x=2$	
		$\frac{d^2y}{dx^2}=6\left(2\right)-15=-3<0$	½
		$\therefore y$ is maximum at $x=2$	
		$y_{\max}=\left(2\right)^3-\frac{15}{2}\left(2\right)^2+18\left(2\right)=14$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	c)	at $x=3$	
		$\frac{d^2y}{dx^2}=6(3)-15=3<0$	½
		$\therefore y$ is minimum at $x=3$	
		$y_{\min}=(3)^3-\frac{15}{2}(3)^2+18(3)=13.5$	½
	d)	A beam is bent in the form of the curve $y=2\sin x-\sin 2x$. Find the radius of curvature of the beam at the point $x=\frac{\pi}{2}$	**04**
	Ans	$y=2\sin x-\sin 2x$	
		$\therefore \frac{dy}{dx}=2\cos x-2\cos 2x$	½
		$\therefore \frac{d^2y}{dx^2}=-2\sin x+4\sin 2x$	½
		$\left(\frac{dy}{dx}\right)_{/\left(x=\frac{\pi}{2}\right)}=2\cos\left(\frac{\pi}{2}\right)-2\cos 2\left(\frac{\pi}{2}\right)=2(0)-2(-1)=2$	½
		$\left(\frac{d^2y}{dx^2}\right)_{/\left(x=\frac{\pi}{2}\right)}=-2\sin\left(\frac{\pi}{2}\right)+4\sin 2\left(\frac{\pi}{2}\right)=-2(1)+4(0)=-2$	½
		$\therefore$ Radius of curvature is $\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	
		$\therefore \rho=\frac{\left[1+(2)^2\right]^{\frac{3}{2}}}{-2}$	1
		$\therefore \rho=-5.59$	1
		$\therefore \rho=5.59$	
3.		**Solve any <u>THREE</u> of the following:**	**12**
	a)	Find the equation of tangent and normal to the curve $2x^2-xy+3y^2=18$ at the point $(3,1)$	**04**
	Ans	$2x^2-xy+3y^2=18$	
		$4x-\left(x\frac{dy}{dx}+y\right)+6y\frac{dy}{dx}=0$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	a)	$4x - x\frac{dy}{dx} - y + 6y\frac{dy}{dx} = 0$	
		$-x\frac{dy}{dx} + 6y\frac{dy}{dx} = -4x + y$	
		$(-x+6y)\frac{dy}{dx} = -4x + y$	
		$\frac{dy}{dx} = \frac{-4x+y}{-x+6y}$	½
		at $(3,1)$	
		Slope of tangent $= \frac{dy}{dx} = \frac{-4(3)+1}{-3+6(1)} = \frac{-11}{3}$	½
		Slope of normal$= \frac{-1}{\frac{dy}{dx}} = \frac{3}{11}$	½
		Equation of tangent	
		$y - y_1 = m(x - x_1)$	
		$y - 1 = \frac{-11}{3}(x-3)$	
		$3y - 3 = -11x + 33$	
		$11x + 3y - 36 = 0$	1
		Equation of normal	
		$y - y_1 = m(x - x_1)$	
		$y - 1 = \frac{3}{11}(x-3)$	
		$11y - 11 = 3x - 9$	
		$3x - 11y + 2 = 0$	1
		---	
	b)	A manufacturer can sell x items at a price of Rs.$(330-x)$ each. The cost of producing x items is Rs. $x^2 + 10x + 12$.Determine the number of items to be sold so that the manufacturer can make the maximum profit.	**04**
	Ans	Selling price of x items $= (330-x)x = 330x - x^2$	
		Cost price of x items = $x^2 + 10x + 12$	
		Profit $=$ Selling price $-$ Cost price	
		Let $P = (330x - x^2) - (x^2 + 10x + 12)$	
		$= 330x - x^2 - x^2 - 10x - 12$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	b)	$P = 320x - 2x^2 - 12$	1
		$\therefore \frac{dP}{dx} = 320 - 4x$	1
		Put $\frac{dP}{dx} = 0$	
		$320 - 4x = 0$	
		$\therefore x = 80$	1
		$\frac{d^2P}{dx^2} = -4 < 0$	1
		$\therefore$ For maximum profit manufacturer can sell 80 items.	
	c)	If $x^y = e^{x-y}$ then prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$	**04**
	Ans	$x^y = e^{x-y}$	
		$\log x^y = \log e^{x-y}$	½
		$y \log x = (x-y)\log e$	½
		$y \log x = x - y$	
		$y \log x + y = x$	
		$y(\log x + 1) = x$	
		$y = \frac{x}{1+\log x}$	1
		$\frac{dy}{dx} = \frac{(1+\log x)(1) - x\left(\frac{1}{x}\right)}{(1+\log x)^2}$	1
		$\frac{dy}{dx} = \frac{1+\log x - 1}{(1+\log x)^2}$	
		$\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$	1
	d)	Evaluate $\int \frac{dx}{2x + x.\log x}$	**04**
	Ans	$I = \int \frac{dx}{2x + x.\log x}$	
		$= \int \frac{dx}{x(2+\log x)}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	d)	Put $2+\log x=t$ OR Put $\log x=t$	½
		$\frac{1}{x}dx=dt$ $\quad$ $\frac{1}{x}dx=dt$	1
		$\therefore I=\int\frac{dt}{t}$ $\quad$ $\therefore I=\int\frac{dt}{2+t}$	½
		$=\log t+c$ $\quad$ $=\log(2+t)+c$	1
		$=\log(2+\log x)+c$ $\quad$ $=\log(2+\log x)+c$	½
4.		**Solve any <u>THREE</u> of the following:**	**12**
	a)	Evaluate : $\int\frac{dx}{x^2+4x+25}$	**04**
	Ans	$I=\int\frac{dx}{x^2+4x+25}$	
		$T.T.=\left(\frac{1}{2}\times Coeff.of.x\right)^2=\left(\frac{1}{2}\times 4\right)^2=4$	1
		$x^2+4x+25=x^2+4x+4-4+25$ OR $I=\int\frac{dx}{x^2+4x-4+4+25}$	1
		$=(x+2)^2+21=(x+2)^2+\left(\sqrt{21}\right)^2$	
		$\therefore I=\int\frac{dx}{(x+2)^2+\left(\sqrt{21}\right)^2}$	1
		$=\frac{1}{\sqrt{21}}\tan^{-1}\left(\frac{x+2}{\sqrt{21}}\right)+c$	1
	b)	Evaluate $\int\frac{dx}{2+3\cos 2x}$	**04**
	Ans	$I=\int\frac{dx}{2+3\cos 2x}$	
		Put $t=\tan x$, $\cos 2x=\frac{1-t^2}{1+t^2}$ and $dx=\frac{dt}{1+t^2}$	1
		$\therefore I=\int\frac{\frac{dt}{1+t^2}}{2+3\left(\frac{1-t^2}{1+t^2}\right)}$	
		$=\int\frac{\frac{dt}{1+t^2}}{\frac{2(1+t^2)+3(1-t^2)}{1+t^2}}$	
		$=\int\frac{dt}{2+2t^2+3-3t^2}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	b)	$=\int \frac{dt}{5-t^2}$	
		$=\int \frac{dt}{\left(\sqrt{5}\right)^2-t^2}$	1
		$=\frac{1}{2\left(\sqrt{5}\right)}\log\left(\frac{\sqrt{5}+t}{\sqrt{5}-t}\right)+c$	1
		$=\frac{1}{2\left(\sqrt{5}\right)}\log\left(\frac{\sqrt{5}+\tan x}{\sqrt{5}-\tan x}\right)+c$	½
		--	
	c)	Evaluate $\int x.\tan^{-1}xdx$	**04**
	Ans	$\int x.\tan^{-1}xdx$	
		$=\int \tan^{-1}x.xdx$	
		$=\tan^{-1}x\int xdx-\int\left(\int xdx\frac{d\left(\tan^{-1}x\right)}{dx}\right)dx$	1
		$=\tan^{-1}x\left(\frac{x^2}{2}\right)-\int\left(\frac{x^2}{2}\right)\frac{1}{1+x^2}dx$	1
		$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\int\frac{x^2}{1+x^2}dx$	
		$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\int\frac{1+x^2-1}{1+x^2}dx$	
		$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\int\left(1-\frac{1}{1+x^2}\right)dx$	1
		$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\left(x-\tan^{-1}x\right)+c$	1
		--	
	d)	Evaluate $\int\frac{x^2+1}{(x+1)(x+2)(x-3)}dx$	**04**
	Ans	$\frac{x^2+1}{(x+1)(x+2)(x-3)}=\frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{x-3}$	½
		$\therefore x^2+1=A(x+2)(x-3)+B(x+1)(x-3)+C(x+1)(x+2)$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	d)	Put $x=-1 \quad \therefore A=\frac{-1}{2}$	½
		Put $x=-2 \quad \therefore B=1$	½
		Put $x=3 \quad \therefore C=\frac{1}{2}$	½
		$\therefore \int \frac{x^2+1}{(x+1)(x+2)(x-3)}dx=\int \frac{\frac{-1}{2}}{x+1}+\frac{1}{x+2}+\frac{\frac{1}{2}}{x-3}dx$	½
		$\therefore \int \frac{x^2+1}{(x+1)(x+2)(x-3)}dx=\frac{-1}{2}\log(x+1)+\log(x+2)+\frac{1}{2}\log(x-3)+c$	½+½+½
		--	**04**
	e)	Evaluate $\int_0^{\pi/2} \frac{dx}{1+\sqrt[3]{\tan x}}$	
	Ans	$\int_0^{\pi/2} \frac{dx}{1+\sqrt[3]{\tan x}}$	
		$=\int_0^{\pi/2} \frac{dx}{1+\sqrt[3]{\frac{\sin x}{\cos x}}}$	
		$=\int_0^{\pi/2} \frac{dx}{1+\frac{\sqrt[3]{\sin x}}{\sqrt[3]{\cos x}}}$	
		$=\int_0^{\pi/2} \frac{dx}{\frac{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}{\sqrt[3]{\cos x}}}$	
		$I=\int_0^{\pi/2} \frac{\sqrt[3]{\cos x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}dx$ ---------------(1)	1
		$=\int_0^{\pi/2} \frac{\sqrt[3]{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt[3]{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt[3]{\sin\left(\frac{\pi}{2}-x\right)}}dx$ --------------By property	
		$I=\int_0^{\pi/2} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}dx$ ---------------(2)	1
		Add (1) and (2)	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	e)	$I+I=\int_0^{\pi/2}\frac{\sqrt[3]{\cos x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}dx+\int_0^{\pi/2}\frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}dx$	
		$2I=\int_0^{\pi/2}\frac{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}dx$	
		$2I=\int_0^{\pi/2}1dx$	1
		$2I=[x]_0^{\pi/2}$	
		$2I=\frac{\pi}{2}$	
		$\therefore I=\frac{\pi}{4}$	1
		--	
5.		**Solve any <u>TWO</u> of the following:**	**12**
	a)	Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ about the x-axis	**06**
	Ans	$\frac{x^2}{9}+\frac{y^2}{4}=1$	
		$\therefore \frac{y^2}{4}=1-\frac{x^2}{9}$	
		$\therefore y^2=\frac{4}{9}\left(9-x^2\right)$	1
		Volume of the solid generated by revolving the ellipse about the x-axis is given by	
		$V=\pi\int_{x=-3}^{x=3}y^2dx$	
		$=\pi\int_{-3}^{3}\frac{4}{9}\left(9-x^2\right)dx$	1
		$=2\pi\int_0^3\frac{4}{9}\left(9-x^2\right)dx \quad \because$ Function is even	1
		$=2\pi\frac{4}{9}\left(9x-\frac{x^3}{3}\right)_0^3$	1
		$=\frac{8\pi}{9}\left[\left(9(3)-\frac{(3)^3}{3}\right)-0\right]$	1
		$=\frac{8\pi}{9}(27-9)$	
		$=16\pi$	1

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Subject Name: Applied Mathematics **Model Answer** Subject Code: 22224

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	b)	Solve the following.	**06**
	(i)	Form the differential equation by eliminating the arbitrary constants if $y^2 = 4ax$	**03**
	Ans	$y^2 = 4ax$ ---------------(1)	
		$2y\frac{dy}{dx} = 4a$ ---------------(2)	1
		Put (2) in (1)	
		$\therefore y^2 = 2y\frac{dy}{dx}x$	1
		$\therefore y = 2x\frac{dy}{dx}$	
		$\therefore 2x\frac{dy}{dx} - y = 0$	1
	(ii)	Solve $\left(1+x^2\right)dy - \left(1+y^2\right)dx = 0$	**03**
	Ans	$\left(1+x^2\right)dy - \left(1+y^2\right)dx = 0$	
		$\left(1+x^2\right)dy = \left(1+y^2\right)dx$	
		$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$	1
		$\therefore$ Solution is,	1
		$\int\frac{dy}{1+y^2} = \int\frac{dx}{1+x^2}$	1
		$\tan^{-1}y = \tan^{-1}x + c$	
	c)	A resistance of 100Ω and inductance of 0.1 henries are connected in series with a battary of 20 volts. find the current in the circuit at any instant , if the relation between L,R and E is $L\frac{di}{dt} + Ri = E$	**06**
	Ans	$L\frac{di}{dt} + Ri = E$	½
		$\therefore \frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$ Comparing with $\frac{dy}{dx} + Py = Q$	
		$\therefore P = \frac{R}{L}$ and $Q = \frac{E}{L}$	
		$IF = e^{\int\frac{R}{L}dt} = e^{\frac{R}{L}t}$	1
		$\therefore$ Solution is $i \cdot IF = \int Q \cdot IF dt + c$	1
		$i \cdot e^{\frac{R}{L}t} = \int\frac{E}{L}e^{\frac{R}{L}t}dt + c$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	$i\cdot e^{\frac{R}{L}t}=\frac{E}{L}\frac{e^{\frac{R}{L}t}}{\frac{R}{L}}+c$	1
		$i\cdot e^{\frac{R}{L}t}=\frac{E}{R}e^{\frac{R}{L}t}+c$	½
		Initially at t = 0 , i = 0 $\therefore$ c = $\frac{-E}{R}$	
		$\therefore i\cdot e^{\frac{R}{L}t}=\frac{E}{R}e^{\frac{R}{L}t}+\left(\frac{-E}{R}\right)$	
		$i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$	1
		When R=100 , L = 0.1 , E= 20	
		$i=\frac{20}{100}\left(1-e^{-\frac{100}{0.1}t}\right)$	
		$i=0.2\left(1-e^{-1000t}\right)$	1
		--	
6.		**Solve any <u>TWO</u> of the following:**	**12**
	a)	Solve the following	**06**
	(i)	Find the approximate root of the equation $x^2+x-3=0$ in the interval $(1,2)$ by using Bisection method(use two iterations)	**03**
	Ans	$x^2+x-3=0$	
		$f(x)=x^2+x-3$	
		$f(1)=-1<0$	
		$f(2)=3>0$	
		root is in $(1,2)$	1
		$\therefore x_1=\frac{1+2}{2}=1.5$	1
		$\therefore f(1.5)=0.75>0$	
		$\therefore$ root is in $(1,1.5)$	
		$\therefore x_2=\frac{1+1.5}{2}=1.25$	1
		OR	
		$x^2+x-3=0$	
		$f(x)=x^2+x-3$	
		$f(1)=-1<0$	
		$f(2)=3>0$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	(a)(i)	root is in $(1,2)$	1

a -ve	b +ve	$x=\frac{a+b}{2}$	$f(x)$	Marks
1	2	1.5	0.75	1
1	1.5	1.25	----------	1

(ii) Solve the following system of the equations by using Gauss elimination method **03**

$x+y+z=6,\ 2x-3y+3z=5,\ 3x+2y-z=4$

Ans

$x+y+z=6$

$2x-3y+3z=5$

$3x+2y-z=4$

$2x+2y+2z=12$ and $3x+3y+3z=18$

$2x-3y+3z=5$ and $3x+2y-z=4$

$-$ and $-$

$5y-z=7$ and $y+4z=14$ 1

$20y-4z=28$

$y+4z=14$

$+$

$21y=42$ ½

$\therefore y=2$ ½

$z=3$ ½

$x=1$ ½

$\therefore$ Solution is $\{1,2,3\}$

---

b) Solve the following system of equations by using Gauss Seidal method (use four iterations) correct upto 3 places of decimals. **06**

$x+7y-3z=-22,\ 5x-2y+3z=18,\ 2x-y+6z=22$

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	b)Ans	$5x-2y+3z=18,\ x+7y-3z=-22,\ 2x-y+6z=22$	1
		$x=\frac{1}{5}(18+2y-3z)$	
		$y=\frac{1}{7}(-22-x+3z)$	1
		$z=\frac{1}{6}(22-2x+y)$	
		Starting with $x_0=y_0=z_0=0$	
		$x_1=3.6$	
		$y_1=-3.657$	1
		$z_1=1.857$	
		$x_2=1.023$	
		$y_2=-2.493$	1
		$z_2=2.910$	
		$x_3=0.857$	
		$y_3=-2.018$	1
		$z_3=3.045$	
		$x_4=0.966$	
		$y_4=-1.976$	1
		$z_4=3.015$	
		---	
	c)	Using Newton-Raphson method find the approximate root of the equation correct upto 3 places of decimals. $x^3-2x-5=0$ (Use four iterations)	**06**
	Ans	$f(x)=x^3-2x-5$	
		$f(2)=-1<0$	
		$f(3)=16>0$	
		Root is in $(2,3)$	1
		$f'(x)=3x^2-2$	1
		Initial root $x_0=2$	
		$\therefore f'(2)=10$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	c)	$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)} = 2.1$	1
		$x_2 = 2.1 - \frac{f(2.1)}{f'(2.1)} = 2.095$	1
		$x_3 = 2.095 - \frac{f(2.095)}{f'(2.095)} = 2.095$	1
		$x_4 = 2.095 - \frac{f(2.095)}{f'(2.095)} = 2.095$	1
		OR	
		$f(x) = x^3 - 2x - 5$	
		$f(2) = -1 < 0$	
		$f(3) = 16 > 0$	
		Root is in $(2,3)$	
		$f'(x) = 3x^2 - 2$	
		Initial root $x_0 = 2$	
		$x_i = \frac{xf'(x) - f(x)}{f'(x)}$	
		$= \frac{x(3x^2-2) - (x^3 - 2x - 5)}{3x^2 - 2}$	
		$= \frac{3x^3 - 2x - x^3 + 2x + 5}{3x^2 - 2}$	
		$= \frac{2x^3 + 5}{3x^2 - 2}$	2
		$x_1 = 2.1$	1
		$x_2 = 2.095$	1
		$x_3 = 2.095$	1
		$x_4 = 2.095$	1
		--	
		Important Note *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.*	

SUMMER– 19 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22224**

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		**Solve any FIVE of the following:**	**10**
	a)	If $f(x)=x^3-5x^2-4x+20$ show that $f(0)=-2f(3)$	**02**
	Ans	$f(x)=x^3-5x^2-4x+20$	
		$\therefore f(0)=(0)^3-5(0)^2-4(0)+20=20$	½
		$\therefore f(3)=(3)^3-5(3)^2-4(3)+20$	½
		$=-10$	
		$\therefore -2f(3)=-2\times -10=20=f(0)$	1
		--	
	b)	State whether the function $f(x)=x^3-3x+\sin x+x\cos x$, is odd or even.	**02**
	Ans	$f(x)=x^3-3x+\sin x+x\cos x$	
		$\therefore f(-x)=(-x)^3-3(-x)+\sin(-x)+(-x)\cos(-x)$	½
		$=-x^3+3x-\sin x-x\cos x$	½
		$=-(x^3-3x+\sin x+x\cos x)$	½
		$=-f(x)$	
		$\therefore$ Given function is odd.	½
		--	
	c)	If $y=\sin x.\cos 2x$, find $\dfrac{dy}{dx}$	**02**
	Ans	$y=\sin x.\cos 2x$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	c)	$\therefore \frac{dy}{dx} = \sin x(-\sin 2x)\times 2 + \cos 2x \cos x$	02
		$= -2\sin x \sin 2x + \cos 2x \cos x$	
	d)	Evaluate: $\int \cos^2 x dx$	**02**
	Ans	$\int \cos^2 x dx = \int \frac{1+\cos 2x}{2} dx$	1
		$= \frac{1}{2}\int (1+\cos 2x) dx$	
		$= \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + c$	1
	e)	Evaluate: $\int \frac{1}{3x+5} dx$	**02**
	Ans	$\int \frac{1}{3x+5} dx$	
		$= \frac{1}{3}\log(3x+5) + c$	02
	f)	Find the area between the the line $y = 2x$,*x-axis* and ordinates $x=1$ to $x=3$.	**02**
	Ans	Area $A = \int_a^b y\ dx$	
		$= \int_1^3 2x dx$	½
		$= 2\left[\frac{x^2}{2}\right]_1^3$ or $\left[x^2\right]_1^3$	½
		$= 2\left[\frac{9}{2} - \frac{1}{2}\right]$ or $\left[3^2 - 1^2\right]$	½
		$= 8$	½
	g)	Find approximate root of the equation $x^2 + x - 3 = 0$ in $(1,2)$ by using Bisection method. $($Use two iterations$)$	**02**

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	g)Ans	Let $f(x)=x^2+x-3$	
		$f(1)=-1$	
		$f(2)=3$	
		$\therefore$ the root is in $(1,2)$	½
		$x_1=\frac{a+b}{2}=\frac{1+2}{2}=1.5$	½
		$f(1.5)=0.75>0$	½
		$x_2=\frac{x_1+a}{2}=\frac{1.5+1}{2}=1.25$	½
		OR	
		Let $f(x)=x^2+x-3$	½
		$f(1)=-1, f(2)=3 \quad \therefore$ the root is in $(1,2)$	

Iteration	a	b	$x=\frac{a+b}{2}$	$f(x)$
I	1	2	1.5	0.75
II	1	1.5	1.25	

Marking: 1½

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.		**Solve any THREE of the following :**	**12**
	a)	Find $\frac{dy}{dx}$ if $x^3+xy^2=y^3+yx^2$	**04**
	Ans	$x^3+xy^2=y^3+yx^2$	
		$x(x^2+y^2)=y(y^2+x^2)$	1
		$x=y$	1
		$\frac{dy}{dx}=1$	2
		OR	
		$x^3+xy^2=y^3+yx^2$	
		$3x^2+2xy\frac{dy}{dx}+y^2=3y^2\frac{dy}{dx}+2xy+x^2\frac{dy}{dx}$	2
		$\frac{dy}{dx}(2xy-3y^2-x^2)=2xy-3x^2-y^2$	1

Q. No.	Sub Q. N.	Answers	Marking Scheme
2.	a)	$\frac{dy}{dx}=\frac{2xy-3x^2-y^2}{2xy-3y^2-x^2}$	1
	b)	Find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$ if $x=a\cos^3\theta, y=b\sin^3\theta$	**04**
	Ans	$x=a\cos^3\theta$	
		$\therefore \frac{dx}{d\theta}=3a\cos^2\theta(-\sin\theta)$	1
		$=-3a\cos^2\theta\sin\theta$	
		$y=b\sin^3\theta$	
		$\therefore \frac{dy}{d\theta}=3b\sin^2\theta\cos\theta$	1
		$=3b\sin^2\theta\cos\theta$	
		$\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	
		$=\frac{3b\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta}$	1
		$=-\frac{b}{a}\tan\theta$	
		$at\ \theta=\frac{\pi}{4}$	
		$\frac{dy}{dx}=-\frac{b}{a}\tan\frac{\pi}{4}$	
		$=-\frac{b}{a}$	1
	c)	A manufacture can sell x items per week at price $(23-0.001x)$ rupees each. It cost $(5x+2000)$ rupees to produce x items Find the number items to be produced eper week for maximum profit.	**04**
	Ans	Let number of item be x	
		Selling price $=(23-0.001x)x$	
		$=23x-0.001x^2$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	c)	Cost price $=\left(5x+2000\right)$	
		profit = selling price − cost price	
		$\therefore\ p=23x-0.001x^2-\left(5x+2000\right)$	½
		$=23x-0.001x^2-5x-2000$	
		$=18x-0.001x^2-2000$	½
		$\dfrac{dp}{dx}=18-0.002x$	½
		$\dfrac{d^2p}{dx^2}=-0.002$	½
		$\therefore$ profit is maximum	
		Let	
		$18-0.002x=0$	½
		$x=\dfrac{18}{0.002}=9000$	1
	d)	Find the radius of curvature of the curve $y=e^x$ at the point where it crosses the Y-axis.	**04**
	Ans	$y=e^x$	
		$\dfrac{dy}{dx}=e^x$	1
		$\dfrac{d^2y}{dx^2}=e^x$	1
		curve crosses Y-axis $\therefore x=0$	
		$\dfrac{dy}{dx}=e^0=1$	½
		$\dfrac{d^2y}{dx^2}=e^0=1$	½
		$\rho=\dfrac{\left(1+\left(\dfrac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$	
		$=\dfrac{\left(1+1^2\right)^{\frac{3}{2}}}{1}=2^{\frac{3}{2}}=2.828$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.		**Solve any <u>THREE</u> of the following**	**12**
	a)	Find equation of tangent and normal to the curve $2x^2 - xy + 3y^2 = 18$ at $(3,1)$	**04**
	Ans	$2x^2 - xy + 3y^2 = 18$	
		$\therefore 4x - \left(x\frac{dy}{dx} + y.1\right) + 6y\frac{dy}{dx} = 0$	1
		$\therefore 4x - x\frac{dy}{dx} - y + 6y\frac{dy}{dx} = 0$	
		$\therefore (6y - x)\frac{dy}{dx} = y - 4x$	½
		$\therefore \frac{dy}{dx} = \frac{y-4x}{6y-x}$	½
		at $(3,1)$	
		$\therefore \frac{dy}{dx} = \frac{1-4(3)}{6(1)-3}$	
		$\therefore \frac{dy}{dx} = \frac{-11}{3}$	
		$\therefore$ slope of tangent , $m = \frac{-11}{3}$	½
		Equation of tangent at $(3,1)$ is	
		$y - 1 = \frac{-11}{3}(x-3)$	
		$\therefore 3y - 3 = -11x + 33$	
		$\therefore 11x + 3y - 36 = 0$	½
		$\therefore$ slope of normal , $m' = \frac{-1}{m} = \frac{3}{11}$	½
		Equation of normal at $(3,1)$ is	
		$y - 1 = \frac{3}{11}(x-3)$	
		$\therefore 11y - 11 = 3x - 9$	
		$\therefore 3x - 11y + 2 = 0$	½
		--	
	b)	Differentiate with respect to x : $x^x + 5^x + x^5 + 5^5$	**04**

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	b)Ans	$y = x^x + 5^x + x^5 + 5^5$	
		Let $u = x^x$	½
		$\log u = \log x^x$	
		$\log u = x \log x$	1
		$\frac{1}{u}\frac{du}{dx} = x.\frac{1}{x} + \log x.1$	
		$\frac{du}{dx} = u(1+\log x) = x^x(1+\log x)$	½
		$\therefore \frac{dy}{dx} = x^x(1+\log x) + 5^x \log 5 + 5x^4$	2
		--	
	c)	If $x^3.y^2 = (x+y)^5$, show that $\frac{dy}{dx} = \frac{y}{x}$	**04**
	Ans	$x^3.y^2 = (x+y)^5$	
		$\log(x^3.y^2) = \log(x+y)^5$	½
		$\log x^3 + \log y^2 = 5\log(x+y)$	½
		$3\log x + 2\log y = 5\log(x+y)$	½
		$3\frac{1}{x} + 2\frac{1}{y}\frac{dy}{dx} = 5\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)$	1
		$\frac{3}{x} + \frac{2}{y}\frac{dy}{dx} = \frac{5}{x+y} + \frac{5}{x+y}\frac{dy}{dx}$	
		$\frac{2}{y}\frac{dy}{dx} - \frac{5}{x+y}\frac{dy}{dx} = \frac{5}{x+y} - \frac{3}{x}$	
		$\frac{dy}{dx}\left(\frac{2}{y} - \frac{5}{x+y}\right) = \frac{5x-3x-3y}{x(x+y)}$	
		$\frac{dy}{dx}\left(\frac{2x+2y-5y}{y(x+y)}\right) = \frac{5x-3x-3y}{x(x+y)}$	½
		$\frac{dy}{dx}\left(\frac{2x-3y}{y}\right) = \frac{2x-3y}{x}$	
		$\frac{dy}{dx} = \frac{y}{x}$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	d)	Evaluate: $\int \frac{e^x(x+1)}{\sin^2(xe^x)}dx.$	**04**
	Ans	$\int \frac{e^x(x+1)}{\sin^2(xe^x)}dx$	
		put $xe^x = t$	
		$(xe^x + e^x.1)dx = dt$	
		$e^x(x+1)dx = dt$	2
		$= \int \frac{dt}{\sin^2 t}$	
		$= \int \cos ec^2 t dt$	½
		$= -\cot t + c$	1
		$= -\cot(xe^x) + c$	½
		--	
4		**Solve any <u>THREE</u> of the following:**	**04**
	a)	Evaluate: $\int \frac{x-3}{x^3-3x^2-16x+48}dx$	
		$\int \frac{x-3}{x^3-3x^2-16x+48}dx$	
		$= \int \frac{x-3}{(x-3)(x-4)(x+4)}dx$	½
		$= \int \frac{dx}{(x-4)(x+4)}$	
		Consider $\frac{1}{(x-4)(x+4)} = \frac{A}{x-4} + \frac{B}{x+4}$	½
		$1 = A(x+4) + B(x-4)$	
		put $x = 4$ $A = \frac{1}{8}$,	½
		put $x = -4$ $B = -\frac{1}{8}$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	a)	$\therefore \int \frac{dx}{(x-4)(x+4)}$ $= \int \left(\frac{\frac{1}{8}}{x-4} + \frac{-\frac{1}{8}}{x+4} \right) dx$ $= \frac{1}{8}\left(\log(x-4) - \log(x+4)\right) + c$	2
	b)	Evaluate : $\int \frac{1}{2+3\cos x} dx$	**04**
	Ans	$\int \frac{1}{2+3\cos x} dx$ Put $\tan\frac{x}{2} = t \quad \cos x = \frac{1-t^2}{1+t^2}, dx = \frac{2dt}{1+t^2}$ $\therefore \int \frac{dx}{2+3\cos x} = \int \frac{1}{2+3\left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{2dt}{1+t^2}$ $= 2\int \frac{1}{5-t^2} dt$ $= 2\int \frac{1}{\left(\sqrt{5}\right)^2 - t^2} dt$ $= 2 \times \frac{1}{2\sqrt{5}} \log\left(\frac{\sqrt{5}+t}{\sqrt{5}-t} \right) + c$ $= \frac{1}{\sqrt{5}} \log\left(\frac{\sqrt{5}+\tan\frac{x}{2}}{\sqrt{5}-\tan\frac{x}{2}} \right) + c$	1 1 ½ 1 ½
	c)	Evalute: $\int e^x . \sin 4x dx$	**04**
	Ans	$\int e^x . \sin 4x dx$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	c)	$=\sin 4x\int e^x dx-\int\left(\int e^x dx\frac{d}{dx}\sin 4x\right).dx$	1
		$=\sin 4xe^x-\int\cos 4x\cdot 4\cdot e^x dx$	½
		$=\sin 4xe^x-4\left[\cos 4x\int e^x dx-\int\left(\int e^x dx\cdot\frac{d}{dx}\cos 4x\right)dx\right]$	1
		$=\sin 4xe^x-4\left[\cos 4xe^x-\int\left(-\sin 4x\cdot 4\cdot e^x\right)dx\right]$	½
		$=\sin 4xe^x-4\left[\cos 4xe^x+4\int\sin 4x\cdot e^x dx\right]$	
		$=\sin 4xe^x-4\cos 4xe^x-16I$	
		$I+16I=\sin 4xe^x-4\cos 4xe^x$	½
		$17I=\sin 4xe^x-4\cos 4xe^x$	
		$I=\frac{1}{17}\left(\sin 4xe^x-4\cos 4xe^x\right)$	½
		--	
	d)	Evaluate: $\int\frac{e^x}{\left(e^x-1\right)\left(e^x+1\right)}dx$	**04**
	Ans	$\int\frac{e^x}{\left(e^x-1\right)\left(e^x+1\right)}dx$	
		put $e^x=t$	
		$e^x dx=dt$	1
		$\int\frac{e^x}{\left(e^x-1\right)\left(e^x+1\right)}dx=\int\frac{dt}{(t-1)(t+1)}$	
		consider $\frac{1}{(t-1)(t+1)}=\frac{A}{t-1}+\frac{B}{t+1}$	½
		$1=A(t+1)+B(t-1)$	
		put $t=1, A=\frac{1}{2}$	½
		put $t=-1, B=-\frac{1}{2}$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	d)	$\frac{1}{(t-1)(t+1)}=\frac{\frac{1}{2}}{t-1}+\frac{-\frac{1}{2}}{t+1}$	
		$\int\frac{dt}{(t-1)(t+1)}=\int\left(\frac{\frac{1}{2}}{t-1}+\frac{-\frac{1}{2}}{t+1}\right)dt$	
		$=\frac{1}{2}\log(t-1)-\frac{1}{2}\log(t+1)+c$	1
		$=\frac{1}{2}\log(e^x-1)-\frac{1}{2}\log(e^x+1)+c$	½
		--	
	e)	Evaluate : $\int_0^{\frac{\pi}{2}}\frac{1}{1+\sqrt{\tan x}}dx$	**04**
	Ans	$I=\int_0^{\frac{\pi}{2}}\frac{1}{1+\sqrt{\tan x}}dx$	
		$=\int_0^{\frac{\pi}{2}}\frac{1}{1+\frac{\sqrt{\sin x}}{\sqrt{\cos x}}}dx$	½
		$I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx---------(1)$	
		by property	
		$\therefore I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}dx$	1
		$=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx---------(2)$	1
		add (1) and (2)	
		$2I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	e)	$2I=\int_0^{\frac{\pi}{2}}1dx$	
		$\therefore 2I=[x]_0^{\frac{\pi}{2}}$	½
		$=\frac{\pi}{2}-0$	
		$I=\frac{\pi}{4}$	½
		OR	
		$I=\int_0^{\frac{\pi}{2}}\frac{1}{1+\sqrt{\tan x}}dx--------(1)$	
		by property	
		$I=\int_0^{\frac{\pi}{2}}\frac{1}{1+\sqrt{\tan\left(\frac{\pi}{2}-x\right)}}dx$	1
		$I=\int_0^{\frac{\pi}{2}}\frac{1}{1+\sqrt{\cot x}}dx$	1
		$\therefore I=\int_0^{\frac{\pi}{2}}\frac{1}{1+\sqrt{\frac{1}{\tan x}}}dx$	½
		$=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\tan x}}{\sqrt{\tan x}+1}dx---------(2)$	
		add (1) and (2)	
		$2I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\tan x}+1}{\sqrt{\tan x}+1}dx$	½
		$2I=\int_0^{\frac{\pi}{2}}1dx$	
		$\therefore 2I=[x]_0^{\frac{\pi}{2}}$	½
		$=\frac{\pi}{2}-0$	
		$I=\frac{\pi}{4}$	½
		--	

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.		**Solve any TWO of the following:**	**12**
	a)	Find the area bounded by two parabolas $y^2 = 2x$ and $x^2 = 2y$.	**06**
	Ans	$y^2 = 2x$ and $x^2 = 2y$ put $y = \frac{x^2}{2}$ in $y^2 = 2x$ $\therefore \left(\frac{x^2}{2}\right)^2 = 2x$ $x^4 - 8x = 0$ $x\left(x^3 - 2^3\right) = 0$ $x = 0, x = 2$ *Let* $y_1 = \sqrt{2x}$, $y_2 = \frac{x^2}{2}$	2
		Area$=\int_a^b \left(y_2 - y_1\right) dx$ $= \int_0^2 \left(\frac{x^2}{2} - \sqrt{2x}\right) dx$	1
		$= \int_0^2 \left(\frac{x^2}{2} - \sqrt{2}x^{\frac{1}{2}}\right) dx$ $= \left[\frac{x^3}{6} - \frac{\sqrt{2}x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2$	1
		$= \frac{2^3}{6} - \frac{2}{3} \times \sqrt{2} \times 2^{\frac{3}{2}} - 0$	1
		$= \frac{4}{3} = 1.333$	1
		--	
5.		**Solve the following:**	**06**
	b) (i)	Form the differential equation from the relation, $y = A.e^x + B.e^{-x}$	**03**

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	Ans	$y = A.e^{x} + B.e^{-x}$	
		$\therefore \frac{dy}{dx} = A.e^{x} - B.e^{-x}$	1
		$\therefore \frac{d^2y}{dx^2} = A.e^{x} + B.e^{-x}$	1
		$\therefore \frac{d^2y}{dx^2} = y$	
		$\therefore \frac{d^2y}{dx^2} - y = 0$	1
		--	
	(ii)	Solve $\frac{dy}{dx} + y\cot x = \cos ecx$	**03**
	Ans	$\frac{dy}{dx} + y\cot x = \cos ecx$	
		$\therefore$ Comparing with $\frac{dy}{dx} + Py = Q$	
		$P = \cot x$, $Q = \cos ecx$	
		Integrating factor $IF = e^{\int \cot x dx}$	
		$= e^{\log(\sin x)}$	
		$= \sin x$	1
		$\therefore \quad y.IF = \int Q.IF dx + c$	
		$\therefore y\sin x = \int \cos ecx.\sin x \, dx$	1
		$\therefore y\sin x = \int 1 \, dx$	
		$\therefore y\sin x = x + c$	1
		--	
	c)	The velocity of a particle is given by $\frac{dx}{dt} = 3t^2 - 6t + 8$. Find distance covered in 2 seconds given that $x = 0$ at $t = 0$	**06**
	Ans	$\frac{dx}{dt} = 3t^2 - 6t + 8$	
		$\therefore dx = \left(3t^2 - 6t + 8\right)dt$	
		$\therefore \int dx = \int \left(3t^2 - 6t + 8\right)dt$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	c)	$\therefore x=\frac{3t^3}{3}-\frac{6t^2}{2}+8t+c$	2
		$\therefore x=t^3-3t^2+8t+c$	
		given that $x=0$ at $t=0$	
		$\therefore c=0$	1
		$\therefore x=t^3-3t^2+8t$	1
		Distance covered in 2 sec,	
		$\therefore x=(2)^3-3(2)^2+8(2)$	
		$\therefore x=12$	1
		--	
6.		**Solve any <u>TWO</u> of the following:**	**12**
	a)(i)	Solve the following system of equations by Jacobi-Iteration method (Two iterations)	**03**
		$15x+2y+z=18$,	
		$2x+20y-3z=19$,	
		$3x-6y+25z=22$	
	Ans	$15x+2y+z=18$,	
		$2x+20y-3z=19$,	
		$3x-6y+25z=22$	
		$x=\frac{1}{15}(18-2y-z)$	
		$y=\frac{1}{20}(19-2x+3z)$	1
		$z=\frac{1}{25}(22-3x+6y)$	
		Starting with $x_0=y_0=z_0=0$	
		$x_1=1.2$	
		$y_1=0.95$	1
		$z_1=0.88$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	a)(i)	$x_2 = 1.015$ $y_2 = 0.962$ $z_2 = 0.964$	1
		--	
	a)(ii)	Solve the following system of equations by using Gauss-Seidal method $\left(\text{Two iterations}\right)$ $5x-2y+3z=18;$ $x+7y-3z=22$, $2x-y+6z=22$	**03**
	Ans	$5x-2y+3z=18;$ $x+7y-3z=22$, $2x-y+6z=22$ $x=\frac{1}{5}\left(18+2y-3z\right)$ $y=\frac{1}{7}\left(22-x+3z\right)$ $z=\frac{1}{6}\left(22-2x+y\right)$	1
		Starting with $x_0 = y_0 = z_0 = 0$ $x_1 = 3.6$ $y_1 = 2.629$ $z_1 = 2.905$	1
		$x_2 = 2.909$ $y_2 = 3.972$ $z_2 = 3.359$	1
		--	
	b)	Solve the following equations by Gauss elimination method. $6x-y-z=19$ $3x+4y+z=26$ $x+2y+6z=22$	**06**

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	b)Ans	$6x-y-z=19$ and $36x-6y-6z=114$ $3x+4y+z=26$ and $x+2y+6z=22$ + ____ + ____ $9x+3y=45$ and $37x-4y=136$ $3x+y=15$ and $37x-4y=136$	1+1
		$12x+4y=60$ $37x-4y=136$ + ____	1
		$49x=196$	
		$\therefore x=4$	1
		$y=3$	1
		$z=2$	1
		***Note:** In the above solution, first x is eliminated and then z is eliminated to find the value of y first. If in case the problem is solved by elimination of another unknown i. e., either first y or z, appropriate marks to be given as per above scheme of marking.*	
		--	
	c)	Using Newton-Raphson method to find the approximate value of $\sqrt[3]{100}$ (perform 4 iterations)	**06**
	Ans	Let $x=\sqrt[3]{100}$	½
		$\therefore x^3-100=0$ $f(x)=x^3-100$	
		$f(4)=-36<0$	½
		$f(5)=25>0$	
		$f'(x)=3x^2$	½
		Initial root $x_0=5$	½
		$\therefore f'(5)=75$	
		$x_1=x_0-\dfrac{f(x_0)}{f'(x_0)}=5-\dfrac{f(5)}{f'(5)}=4.6667$	1
		$x_2=4.6667-\dfrac{f(4.6667)}{f'(4.6667)}=4.6417$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	c)	$x_3 = 4.6417 - \frac{f(4.6417)}{f'(4.6417)} = 4.6416$	1
		$x_4 = 4.6416 - \frac{f(4.6416)}{f'(4.6416)} = 4.6416$	1
		OR	
		$Let\ f(x) = x^3 - 100$	½
		$f(4) = -36 < 0 \quad f(5) = 25 > 0$	½
		$f'(x) = 3x^2$	½
		Initial root x_0=5	½
		$x_i = x - \frac{f(x)}{f'(x)} = x - \frac{x^3-100}{3x^2}$ $= \frac{3x^3 - x^3 + 100}{3x^2}$ $= \frac{2x^3+100}{3x^2}$	2
		$x_1 = 4.6667$	½
		$x_2 = 4.6417$	½
		$x_3 = 4.6416$	½
		$x_4 = 4.6416$	½
		--	
		Important Note	
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.	
		--	

WINTER– 19 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22224**

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		**Solve any FIVE of the following:**	**10**
	a)	State whether the function is odd or even, $f(x)=\dfrac{e^x+e^{-x}}{2}$	**02**
	Ans	$f(x)=\dfrac{e^x+e^{-x}}{2}$	
		$\therefore f(-x)=\dfrac{e^{-x}+e^{-(-x)}}{2}$	½
		$\therefore f(-x)=\dfrac{e^{-x}+e^{x}}{2}$	½
		$\therefore f(-x)=f(x)$	½
		$\therefore$ function is even.	½
		--	
	b)	If $f(x)=\log_4 x+3$, find $f\left(\dfrac{1}{4}\right)$	**02**
	Ans	$f(x)=\log_4 x+3$	
		$f\left(\dfrac{1}{4}\right)=\log_4\left(\dfrac{1}{4}\right)+3$	1
		$=-\log_4 4+3$	
		$=-1+3=2$	1

WINTER– 19 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22224**

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	c)	Find $\frac{dy}{dx}$ if $y = x^2.e^x$	**02**
	Ans	$y = x^2.e^x$	
		$\frac{dy}{dx} = x^2.e^x + e^x.2x$	1
		$\frac{dy}{dx} = xe^x\left(x+2\right)$	1
		--	
	d)	Evaluate $\int\left[e^x + a^x + x^a + a^a\right]\,dx$	**02**
	Ans	$\int\left[e^x + a^x + x^a + a^a\right]\,dx$	
		$= e^x + \frac{a^x}{\log a} + \frac{x^{a+1}}{a+1} + a^a x + c$	2
		--	
	e)	Evaluate: $\int\left[\frac{1}{1+\cos 2x}\right]dx$	**02**
	Ans	$\int\left[\frac{1}{1+\cos 2x}\right]dx$	
		$= \int\left[\frac{1}{2\cos^2 x}\right]dx$	1
		$= \frac{1}{2}\int \sec^2 x dx$	
		$= \frac{1}{2}\tan x + c$	1
		--	
	f)	Find the area bounded by $y = x$, X − axis and $x = 0$ to $x = 4$.	**02**
	Ans	Area $A = \int_a^b y\ dx$	
		$= \int_0^4 xdx$	½
		$= \left[\frac{x^2}{2}\right]_0^4$	½

WINTER– 19 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22224**

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		$=\left(\frac{4^2}{2}-0\right)$	½
		$=8$	½
		--	
	g)	Find a real root of the equation $x^3+4x-9=0$ in the interval $(1,2)$ by using Bisection method. (only one iteration)	**02**
	Ans	Let $f(x)=x^3+4x-9$	
		$f(1)=-4$	1
		$f(2)=7$	
		$\therefore$ the root is in $(1,2)$	
		$x_1=\frac{a+b}{2}=\frac{1+2}{2}=1.5$	1
		--	
2		**Solve any <u>THREE</u> of the following:**	**12**
	a)	Find $\frac{dy}{dx}$, if $y=\frac{5e^x}{3e^x+1}$ at $x=0$	**04**
	Ans	$y=\frac{5e^x}{3e^x+1}$	
		$\frac{dy}{dx}=\frac{(3e^x+1)5e^x-5e^x(3e^x)}{(3e^x+1)^2}$	2
		$\frac{dy}{dx}=\frac{15e^{2x}+5e^x-15e^{2x}}{(3e^x+1)^2}$	
		$\frac{dy}{dx}=\frac{5e^x}{(3e^x+1)^2}$	1
		$at\ x=0$	
		$\frac{dy}{dx}=\frac{5e^0}{(3e^0+1)^2}$	
		$=\frac{5}{16}$ or 0.3125	1
		--	

WINTER– 19 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22224**

Q. No.	Sub Q. N.	Answers	Marking Scheme
2.	b)	If $x=a\left(1+\cos\theta\right)$, $y=a\left(1-\cos\theta\right)$, find $\dfrac{dy}{dx}$	**04**
	Ans	$x=a\left(1+\cos\theta\right)$, $y=a\left(1-\cos\theta\right)$	
		$\dfrac{dx}{d\theta}=a\left(-\sin\theta\right)=-a\sin\theta$, $\dfrac{dy}{d\theta}=a\left(0+\sin\theta\right)=a\sin\theta$	1+1
		$\dfrac{dy}{dx}=\dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\dfrac{a\sin\theta}{-a\sin\theta}$	1
		$\dfrac{dy}{dx}=-1$	1
		--	
	c)	A metal wire 36 cm long is bent to form a rectangle. Find its dimensions when its area is maximum.	**04**
	Ans	Let length of rectangle $=x$, breadth $=y$	
		$\therefore 2x+2y=36$	
		$\therefore y=18-x$	1
		Area $A=x\times y$	
		$A=x\left(18-x\right)$	1
		$\therefore A=18x-x^2$	
		$\therefore \dfrac{dA}{dx}=18-2x$	½
		$\therefore \dfrac{d^2A}{dx^2}=-2$	½
		Let $\dfrac{dA}{dx}=0$	
		$\therefore 18-2x=0$	
		$\therefore x=9$	½
		at $x=9$	
		$\dfrac{d^2A}{dx^2}=-2<0$	
		Area is maximum at $x=9$	
		Length $=9$; breadth $=9$	½
		--	

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	d)	Find radius of curvature of a curve $y=\log(\sin x)$ at $x=\pi/2$	**04**
	Ans	$y=\log(\sin x)$	
		$\therefore \frac{dy}{dx}=\frac{1}{\sin x}.\cos x=\cot x$	½
		$\therefore \frac{d^2y}{dx^2}=-\operatorname{cosec}^2 x$	½
		at $x=\pi/2$	
		$\frac{dy}{dx}=\cot\frac{\pi}{2}=0$	½
		$\frac{d^2y}{dx^2}=-\operatorname{cosec}^2\frac{\pi}{2}=-1$	½
		$\therefore$ Radius of curvature is, $\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	
		$\therefore \rho=\frac{\left[1+(0)^2\right]^{\frac{3}{2}}}{-1}$	1
		$\therefore \rho=-1$ i.e. 1	1
		--	
3.		**Solve any <u>THREE</u> of the following:**	**12**
	a)	Find equation of the tangent and normal to the curve $4x^2+9y^2=40$ at point $(1,2)$	**04**
	Ans	$4x^2+9y^2=40$	
		$\therefore 8x+18y\frac{dy}{dx}=0$	½
		$\therefore \frac{dy}{dx}=\frac{-8x}{18y}$	
		$\therefore \frac{dy}{dx}=\frac{-4x}{9y}$	½
		at $(1,2)$	
		$\therefore \frac{dy}{dx}=\frac{-4(1)}{9(2)}$	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	a)	$\therefore \frac{dy}{dx}=\frac{-2}{9}$	
		$\therefore$ slope of tangent , $m=\frac{-2}{9}$	½
		Equation of tangent at $(1,2)$ is	
		$y-2=\frac{-2}{9}(x-1)$	½
		$\therefore 9y-18=-2x+2$	
		$\therefore 2x+9y-20=0$	½
		$\therefore$ slope of normal , $m'=\frac{-1}{m}=\frac{9}{2}$	½
		Equation of normal at $(1,2)$ is	
		$y-2=\frac{9}{2}(x-1)$	½
		$\therefore 2y-4=9x-9$	
		$\therefore 9x-2y-5=0$	½
		--	
	b)	Find $\frac{dy}{dx}$ if $y=\tan^{-1}\left[\frac{2x}{1+35x^2}\right]$	**04**
	Ans	$y=\tan^{-1}\left[\frac{7x-5x}{1+7x.5x}\right]$	1
		$y=\tan^{-1}7x-\tan^{-1}5x$	1
		$\frac{dy}{dx}=\frac{7}{1+49x^2}-\frac{5}{1+25x^2}$	2
		--	
	c)	If $x^y=e^{x-y}$ Show that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$	**04**
	Ans	$x^y=e^{x-y}$	
		$\log x^y=\log e^{x-y}$	½
		$y\log x=x-y\log e$	½
		$y\log x=x-y$	
		$y\log x+y=x$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	c)	$y\left(\log x+1\right)=x$	
		$y=\dfrac{x}{\log x+1}$	1
		$\dfrac{dy}{dx}=\dfrac{\left(\log x+1\right).1-x.\dfrac{1}{x}}{\left(\log x+1\right)^2}$	1
		$=\dfrac{\log x}{\left(\log x+1\right)^2}$	½
		--	
	d)	Evaluate $\int\dfrac{dx}{5+3\cos 2x}$	**04**
	Ans	$\int\dfrac{dx}{5+3\cos 2x}$	
		Put $\tan x=t$, $dx=\dfrac{dt}{1+t^2}$	
		$\cos 2x=\dfrac{1-t^2}{1+t^2}$	
		$\int\dfrac{\dfrac{dt}{1+t^2}}{5+3\left(\dfrac{1-t^2}{1+t^2}\right)}$	1
		$=\int\dfrac{dt}{5\left(1+t^2\right)+3\left(1-t^2\right)}$	
		$=\int\dfrac{dt}{5+5t^2+3-3t^2}$	
		$=\int\dfrac{dt}{2t^2+8}$	1
		$=\int\dfrac{dt}{\left(\sqrt{2}t\right)^2+\left(\sqrt{8}\right)^2}$ OR $=\dfrac{1}{2}\int\dfrac{dt}{t^2+4}$	
		$=\dfrac{1}{\sqrt{8}}\tan^{-1}\left(\dfrac{\sqrt{2}t}{\sqrt{8}}\right).\dfrac{1}{\sqrt{2}}+c$ $=\dfrac{1}{2}.\dfrac{1}{2}\tan^{-1}\left(\dfrac{t}{2}\right)+c$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	d)	$=\frac{1}{\sqrt{16}}\tan^{-1}\left(\frac{\sqrt{2}\tan x}{\sqrt{8}}\right)+c$ OR $=\frac{1}{4}\tan^{-1}\left(\frac{t}{2}\right)+c$	½
		$=\frac{1}{4}\tan^{-1}\left(\frac{\tan x}{2}\right)+c$ OR $=\frac{1}{4}\tan^{-1}\left(\frac{\tan x}{2}\right)+c$	½
		--	
4.		**Solve any <u>THREE</u> of the following:**	**12**
	a)	Evaluate $\int\frac{\left[e^x\left(x+1\right)\right]}{\cos^2\left(x.e^x\right)}\,dx$	**04**
	Ans	$\int\frac{\left[e^x\left(x+1\right)\right]}{\cos^2\left(x.e^x\right)}\,dx$	
		Put $x.e^x=t$	
		$\therefore\left(x.e^x+e^x.1\right)dx=dt$	
		$\left[e^x\left(x+1\right)\right]dx=dt$	1
		$\therefore\int\frac{1}{\cos^2 t}\,dt$	1
		$=\int \sec^2 t\,dt$	1
		$=\tan t+c$	½
		$=\tan\left(x.e^x\right)+c$	½
		--	
	b)	Evaluate : $\int\frac{dx}{2x^2+3x+2}$	**04**
	Ans	$\int\frac{dx}{2x^2+3x+2}$	
		$=\frac{1}{2}\int\frac{dx}{x^2+\frac{3}{2}x+1}$	½
		$==\frac{1}{2}\int\frac{dx}{x^2+\frac{3}{2}x+\frac{9}{16}+1-\frac{9}{16}}$	1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	b)	$=\frac{1}{2}\int\frac{dx}{\left(x+\frac{3}{4}\right)^2+\frac{7}{16}}$	½
		$=\frac{1}{2}\int\frac{dx}{\left(x+\frac{3}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2}$	1
		$=\frac{1}{2}\frac{1}{\frac{\sqrt{7}}{4}}\tan^{-1}\left(\frac{x+\frac{3}{4}}{\frac{\sqrt{7}}{4}}\right)+c$	1
		$=\frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{4x+3}{\sqrt{7}}\right)+c$	
	c)	Evaluate $\int x^2.\tan x\ dx$	**04**
	Ans	$\int x^2.\tan x\ dx$	
		$=x^2\left(\int\tan x dx\right)-\int\left(\int\tan x dx.\frac{d}{dx}\left(x^2\right)\right)dx$	½
		$=x^2\log\left(\sec x\right)-\int\log\left(\sec x\right).2x\ dx$	1
		$=x^2\log\left(\sec x\right)-2\left[\log\left(\sec x\right)\frac{x^2}{2}-\int\frac{1}{\sec x}.\sec x\tan x.\frac{x^2}{2}dx\right]$	½
		$=x^2\log\left(\sec x\right)-2\left[\log\left(\sec x\right)\frac{x^2}{2}-\frac{1}{2}\int x^2.\tan x.dx\right]$	
		$=x^2\log\left(\sec x\right)-2\left[\log\left(\sec x\right)\frac{x^2}{2}-\frac{1}{2}I\right]$	½
		$I=x^2\log\left(\sec x\right)-\log\left(\sec x\right)x^2+I$	½
		<u>Note</u>: If students attempted to solve the question give appropriate marks.	
	d)	Evaluate $\int\frac{\sec^2 x}{\left(\tan x\right)\left(\tan x+1\right)}dx$	**04**

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	Ans	$\int \frac{\sec^2 x}{(\tan x)(\tan x+1)}dx$	
		Put $\tan x = t$	1
		$\therefore \sec^2 x\ dx = dt$	
		$\therefore \int \frac{1}{(t)(t+1)}dt$	
		$\frac{1}{(t)(t+1)} = \frac{A}{t}+\frac{B}{t+1}$	
		$\therefore 1 = A(t+1)+B(t)$	½
		$\therefore$ Put $t=0$, $A=1$	½
		Put $t=-1$, $B=-1$	
		$\therefore \frac{1}{(t)(t+1)} = \frac{1}{t}-\frac{1}{t+1}$	½
		$\therefore \int \frac{1}{(t)(t+1)}dt = \int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt$	
		$=\log(t)-\log(t+1)+c$	1
		$=\log(\tan x)-\log(\tan x+1)+c$	½
		--	
	e)	Evaluate: $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}}dx$	**04**
	Ans	$\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}}dx$	
		$=\int_0^{\frac{\pi}{2}} \frac{1}{1+\frac{\sqrt{\sin x}}{\sqrt{\cos x}}}dx$	
		Let I= $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ --------------------(1)	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	e)	$I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}dx$	1
		$I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ----------------------------(2)	1
		Add (1) and (2)	
		$I+I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx+\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$	1
		$2I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$	
		$2I=\int_0^{\frac{\pi}{2}} dx$	
		$2I=\left[x\right]_0^{\frac{\pi}{2}}$	½
		$2I=\frac{\pi}{2}-0$	
		$\therefore I=\frac{\pi}{4}$	½

5		**Solve any <u>TWO</u> of the following:**	**12**
	a)	Find area bounded by the curve $y=x^2$ and the line $y=x$	**06**
	Ans	We have $y=x^2$ and $y=x$	
		$\therefore x^2-x=0$	
		$\therefore x(x-1)=0$	
		$\therefore x=0$ or $x=1$	1
		Area $=\int_a^b (y_1-y_2)dx$	
		$=\int_0^1 (x^2-x)dx$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	a)	$=\left[\frac{x^3}{3}-\frac{x^2}{2}\right]_0^1$	1
		$=\left[\frac{1^3}{3}-\frac{1^2}{2}-0\right]$	1
		$=-\frac{1}{6}$	1
		$\therefore A=\frac{1}{6}$ or 0.167 $(\because$ Area is always $+ve)$	1
		--	
	b)	Attempt the following:	**06**
	i)	From the differential equation by eliminating the arbitray constant if $y=A\cos x+B\sin x.$	**03**
	Ans	$y=A\cos x+B\sin x.$	
		$\frac{dy}{dx}=-A\sin x+B\cos x$	1
		$\frac{d^2y}{dx^2}=-A\cos x-B\sin x$	
		$=-\left(A\cos x+B\sin x\right)$	1
		$=-y$	
		$\frac{d^2y}{dx^2}+y=0$	1
		--	
	ii)	Solve $\left(1+x^2\right)dy-x^2.ydx=0$	**03**
	Ans	$\left(1+x^2\right)dy-x^2.ydx=0$	
		$\left(1+x^2\right)dy=x^2.ydx$	
		$\frac{dy}{y}=\frac{x^2dx}{1+x^2}$	
		$\int\frac{dy}{y}=\int\frac{x^2dx}{1+x^2}$	1
		$\int\frac{dy}{y}=\int\frac{1+x^2-1dx}{1+x^2}$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.		$\int \frac{dy}{y} = \int \left[1 - \frac{1}{1+x^2}\right] dx$	1
		$\log y = x - \tan^{-1} x + c$	1
		--	
	c)	Solve the D.E $\frac{dq}{dt} + \frac{1}{RC} q = \frac{E}{R}$ given that $q = 0$ when $t = 0$ and E,R,C are constant	**06**
	Ans	$\frac{dq}{dt} + \frac{1}{RC} q = \frac{E}{R}$	
		$I.F = e^{\int \frac{1}{RC} dt}$	
		$= e^{\frac{t}{RC}}$	1
		$\therefore q.e^{\frac{t}{RC}} = \int \frac{E}{R} . e^{\frac{t}{RC}} dt$	1
		$= \frac{E}{R} e^{\frac{t}{RC}} . \frac{1}{\frac{1}{RC}} + c_1$	1
		$q.e^{\frac{t}{RC}} = e^{\frac{t}{RC}} EC + c_1$	
		given that $q = 0$ when $t = 0$	
		$0 = e^0 EC + c_1$	1
		$c_1 = -EC$	1
		$q.e^{\frac{t}{RC}} = e^{\frac{t}{RC}} EC - EC$	
		$q = EC\left(1 - e^{-\frac{t}{RC}}\right)$	1
		--	
6.		**Solve any <u>TWO</u> of the following:**	**12**
	a)	Attempt the following:	**06**
	i)	Solve the equations by Gauss-Seidal method. (two iterations only)	**03**
		$10x + y + 2z = 13$, $3x + 10y + z = 14$, $2x + 3y + 10z = 15$	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	a)(i)	$10x+y+2z=13$, $3x+10y+z=14$, $2x+3y+10z=15$ $x=\frac{1}{10}(13-y-2z)$ $y=\frac{1}{10}(14-3x-z)$ $z=\frac{1}{10}(15-2x-3y)$ Starting with $x_0=y_0=z_0=0$ $x_1=1.3$ $y_1=1.01$ $z_1=0.937$ $x_2=1.012$ $y_2=1.003$ $z_2=0.997$	1 1 1
	(ii)	Solve the following system of equation by using Jacobi-Iteration method. (two iterations) $5x+2y+z=12$, $x+4y+2z=15$, $x+2y+5z=20$	**03**
	Ans	$5x+2y+z=12$ $x+4y+2z=15$ $x+2y+5z=20$ $x=\frac{1}{5}(12-2y-z)$ $y=\frac{1}{4}(15-x-2z)$ $z=\frac{1}{5}(20-x-2y)$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	a)(ii)	Starting with $x_0 = y_0 = z_0 = 0$	
		$x_1 = 2.4$ $y_1 = 3.75$ $z_1 = 4$	1
		$x_2 = 0.1$ $y_2 = 1.15$ $z_2 = 2.02$	1
		--	
	b)	Solve the following system of equations by using Gauss elimination method. $x+2y+3z=14, 3x+y+2z=11, 2x+3y+z=11$	**06**
	Ans	$3x+6y+9z=42$ $3x+y+2z=11$ $-$ ______ $5y+7z=31$ and $2x+4y+6z=28$ $2x+3y+z=11$ $-$ ______ $y+5z=17$	1+1
		$5y+25z=85$ $5y+7z=31$ $-$ ______	1
		$18z=54$	1
		$\therefore z=3$	1
		$y=2$ $x=1$	1
		--	
	c)	Using Newton-Raphson method find the approximate root of the equation $x^2+x-5=0$ (use four iterations)	**06**
	Ans	$f(x)=x^2+x-5$ $f(1)=-3<0$ $f(2)=1>0$	1
		$f'(x)=2x+1$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	c)	Initial root $x_0 = 2$	
		$\therefore f'(2) = 5$	
		$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)} = 1.8$	1
		$x_2 = 1.8 - \frac{f(1.8)}{f'(1.8)} = 1.7913$	1
		$x_3 = 1.7913 - \frac{f(1.7913)}{f'(1.7913)} = 1.7912$	1
		$x_4 = 1.7912 - \frac{f(1.7912)}{f'(1.7912)} = 1.7912$	1
		OR	
		Let $f(x) = x^2 + x - 5$	
		$f(1) = -3 < 0 \quad f(2) = 1 > 0$	1
		$f'(x) = 2x + 1$	1
		Initial root $x_0 = 2$	
		$x_i = x - \frac{f(x)}{f'(x)} = x - \frac{x^2 + x - 5}{2x + 1}$	
		$= \frac{2x^2 + x - x^2 - x + 5}{2x + 1}$	
		$= \frac{x^2 + 5}{2x + 1}$	2
		$x_1 = 1.8$	½
		$x_2 = 1.7913$	½
		$x_3 = 1.7912$	½
		$x_4 = 1.7912$	½
		--	
		Important Note	
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.	